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An approximate theory of gun tunnel behaviour

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Approximate methods are used in a theoretical investigation of piston acceleration, peak pressure and piston oscillations in a gun tunnel. Behaviour in all these respects, at a given ratio of initial driver and test gas pressures, is related to the value of a 'gun tunnel parameter', Λ, and the ratio of the speeds of sound in driver and test gases. It is shown that large values of Λ must be combined with an enhanced speed of sound in the driver if high performance is to be achieved with manageable peak pressures. With air as test gas, using values of Λ achieved in current practice and a suitable driver gas, the analysis indicates that piston speeds approaching four times the speed of sound in air may be obtained.

1. Introduction

Gun tunnel operation is one of three ways of using gas compression by motion of a free piston as a means of generating high-velocity flow, the other two being the free-piston shock tube, and the free-piston shock tunnel. All utilize the same basic facility and, indeed, appear to offer alternative modes of operation for the same apparatus. They differ in that the gun tunnel uses the free piston to directly compress the test gas before ejecting it through a hypersonic nozzle, whilst in the other two it is used to compress and heat a secondary driver gas, which is then employed to operate a shock tube containing the test gas. The virtue of the gun tunnel is that it will produce a moderately high-velocity flow for many milliseconds, whilst the other two produce flow at a much higher velocity, but generally for a time which is shorter by at least an order of magnitude. In the present paper an analysis of gun tunnel behaviour is developed, and used to predict approximately the limits of gun tunnel performance. It is intended to present separate papers on the free-piston shock tube and shock tunnel.

The function of a gun tunnel is to increase the temperature of a given mass of gas, contained within a particular compression tube volume at a fixed initial temperature (usually room temperature), by raising it to a given pressure. In the absence of any entropy change, the temperature increase is independent of the process of compression, depending only on the initial and final pressures. Use of a high-speed piston magnifies this temperature increase by giving rise to shock waves, with associated entropy increases, in the test gas. Because these entropy changes themselves increase with piston speed, the maximum piston velocity produced will be taken as a measure of performance. A method for calculating the final temperature corresponding to a given piston speed is outlined

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by Winter (1960), where it can be seen that for piston speeds above about twice the speed of sound in the test gas, and a fixed ratio of initial and final pressures, the final ideal gas temperature becomes roughly proportional to the piston speed.

Analyses relevant to gun tunnel behaviour do exist. In particular, both Meyer (1957) and Winter (1960) have considered the accelerating part of the piston motion, whilst in addition Winter has treated those features which are most important in gun tunnel operation, including the reversal of piston motion at the end of the primary compression stroke, the peak pressures produced, and the final temperatures in the compressed test gas. However, in both cases numerical procedures are involved and, in the latter case especially, a great deal of labour is called for in treating any particular example. Thus, whilst these methods are proper and necessary for an exact calculation of piston and flow behaviour in a specific case, they are not suitable to a broad quantitative study of the influence on gun tunnel behaviour of such factors as the speed of sound in the driver gas. For such a purpose it is more convenient to develop and use approximate treatments.

After outlining the initial assumptions made for the study, consideration is first given to the accelerating phase of the motion. During the later stages of this phase the piston asymptotically approaches a limiting velocity, and the motion is therefore treated by regarding the piston velocity as a perturbation from this limit. For this treatment a value of the perturbation velocity is required at a suitable initial point, and this is obtained by regarding the early stages of the piston motion as a perturbation from the motion of a piston accelerating into a vacuum. Then attention is given to the peak pressure developed in the test gas between the piston and the end of the tube as the piston reverses its motion on completion of the primary compression stroke and, establishing a first-order theory, the approximate piston velocities associated with given ratios of peak pressure to driver pressure are derived. Finally, the oscillating motion of the piston before it comes to rest is investigated, with particular emphasis on the first cycle. Using air as driver gas, this phase of the motion is completed in a time short compared with the period of constant pressure which follows, and has therefore received little attention to date; but as the speed of sound in the driver gas increases it is seen to become more important.

2. Initial assumptions

The driver section and compression tube of the gun tunnel are, for convenience, assumed to be of identical cross-section. If desired, the effect of a reduction of area in passing from the one to the other may be accounted for by adjusting driver pressure and speed of sound to appropriate equivalent values (e.g. Alpher & White 1958). In particular, use of a driver of infinite cross-sectional area would involve taking equivalent values $(\frac{1}{2}\gamma+\frac{1}{2})^{\gamma/(\gamma-1)}$ times the actual value for the pressure, and $(\frac{1}{2}\gamma+\frac{1}{2})^{\frac{1}{2}}$ times the actual value for the speed of sound. Cox & Winter (1961) have pointed out that this procedure is strictly correct only when the piston is locally supersonic with respect to the driver gas, and will overestimate the pressure and speed of sound at the rear face of the piston when it is locally subsonic. Since the emphasis in the present paper is on conditions under which the piston reaches velocities that are locally supersonic, the influence of this error need only be considered in relation to the accelerating phase of the motion. Even then, it is not expected to be important in the present context. For example, calculation of the motion of a piston accelerating into a vacuum with a driver gas of $\gamma = 1{\cdot}4$ indicates that, using a more accurate expression for the pressure behind the piston provided by Cox & Winter, the distance the piston travels before achieving a velocity equal to the ambient speed of sound in the driver is increased by only 8 %. Thus the effect on the velocity of a piston accelerating into a vacuum is small. In addition, since the variation of this velocity with distance along the tube governs the perturbation to the velocity due to the presence of test gas before the piston (see § 3), the effect on this perturbation may also be expected to be small.

The assumption of ideal gases is maintained throughout this study. In addition, and following Meyer, and Winter, the effects of gas leakage past the moving piston, friction between the piston and the compression tube, boundary-layer formation, and heat transfer to the walls of the compression tube, have all been neglected in the analysis. It is expected that the first two of these effects can, if necessary, be sufficiently reduced by suitable piston design. The last two, in experiments to date, have not obviously influenced those aspects of the piston behaviour considered here—see, for example, Bray, Pennelegion & East 1959; Smith 1960; Cox & Winter 1961; Stalker 1961. The last-named experiments were conducted at relatively low pressures ($P_R < 75$ p.s.i.) with a compression tube 0·25 in. in diameter, when these two effects, if important at larger scale, would be expected to be particularly pronounced.

3. Accelerating phase

Figure 1 shows the gun tunnel as the piston is accelerating along the compression tube. The driver section is presumed to be of a length such that the expansion wave reflected from the closed end of the driver does not influence the piston motion. Then, remembering that the driver and compression tube are of identical cross-section, it follows that an unsteady simple wave exists between the piston and the undisturbed driver gas, and the pressure at the rear face of the piston is given by

$$P_D = P_R\{1-\tfrac{1}{2}(\gamma-1)\,v/a_R\}^{2\gamma/(\gamma-1)},$$

where P_R and a_R are the pressure and speed of sound in the undisturbed driver gas, v the instantaneous piston velocity, and γ the ratio of specific heats in the driver gas. The equation of motion for the piston can therefore be written as

$$\frac{dv}{dt} = \frac{P_R}{\sigma}\left\{\left[1-\tfrac{1}{2}(\gamma-1)\frac{v}{a_R}\right]^{2\gamma/(\gamma-1)} - \frac{P_0}{P_R}\frac{P_S}{P_0}\right\}, \tag{1}$$

where P_S is the instantaneous pressure at the forward face of the piston, P_0 the initial pressure in the test gas, and σ the piston mass per unit cross-sectional area.

The pressure P_S is generated through a shock and subsequent compression wave preceding the accelerating piston, but, as Winter (1960) has pointed out, its value is well represented by the pressure obtained using the instantaneous velocity in the normal shock equations. Since gun tunnels have been used almost exclusively to compress air or nitrogen the ratio of specific heats in the test gas has, for convenience, been taken as 1·4. A further simplifying approximation is then made, and the relation

$$P_S = P_0\left\{1{\cdot}68\left(\frac{v}{a_0}\right)^2+1\right\} \tag{2}$$

is used, where a_0 is the initial speed of sound in the test gas. This expression is at its most inaccurate at low values of v/a_0, yielding a value for P_S which is roughly 25 % below that predicted by the exact shock relations when $v/a_0 = 1$, and 13 % below when $v/a_0 = 2$. The approximation can be greatly improved by adding a

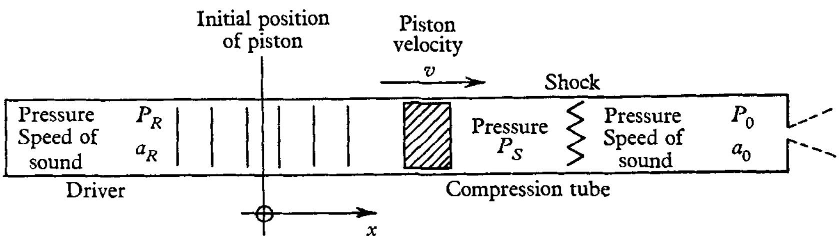

FIGURE 1. Constant-area gun tunnel.

term linear in v/a_0, which can be used to obtain agreement with the shock relations to within a few percent. However, when this added term is included in the analysis, a change of only a few percent in the perturbation velocity is obtained, and it is concluded that the simple relation of equation (2) is an adequate approximation.

The piston approaches a limiting velocity, Ua_R, which is identical with the velocity of the interface in a constant area shock tube using the same gases, and is characterized by equal pressures at the front and rear faces of the piston. Letting $v = (U+u)\,a_R$, $|u| \ll 1$ as the piston approaches this limiting velocity, whence, using (2), equation (1) can be solved to yield

$$u = u_0 \exp\{A(x_0-x)\}, \tag{3}$$

where x is the distance the piston has travelled along the tube, u_0 and x_0 are convenient simultaneous values of u and x,

$$A = \frac{P_R}{\sigma a_R^2}\frac{1}{U}\left[\gamma\{1-\tfrac{1}{2}(\gamma-1)\,U\}^{(\gamma+1)/(\gamma-1)}+\frac{P_0}{P_R}3{\cdot}36U\left(\frac{a_R}{a_0}\right)^2\right],$$

and P_0/P_R is in this instance given approximately as

$$\frac{P_0}{P_R} = \{1-\tfrac{1}{2}(\gamma-1)\,U\}^{2\gamma/(\gamma-1)}\Big/\left[1{\cdot}68\left(\frac{a_R}{a_0}\right)^2U^2+1\right]$$

Specification of the piston velocity therefore rests on determination of u_0 and x_0 in (3). In the early stages of the piston motion, when the piston velocity is low, the pressure at the front face of the piston plays a very small part in determining the piston trajectory, suggesting that the velocity at any point is only a little different from the velocity, Va_R, associated with a piston accelerating into a vacuum. Thus, combining (1) with (2), and putting $v = (V+u')\,a_R$, it is found that

$$\frac{du'}{dV}\{1-\tfrac{1}{2}(\gamma-1)\,V\}^{2\gamma/(\gamma-1)} = -\gamma u'\{1-\tfrac{1}{2}(\gamma-1)\,V\}^{(\gamma+1)/(\gamma-1)} - \frac{P_0}{P_R}\left\{1{\cdot}68\left(\frac{a_R}{a_0}\right)^2 V^2+1\right\}+\epsilon, \tag{4}$$

where

$$\epsilon = -\frac{P_0}{P_R}\left\{1{\cdot}68(2u'V+u'^2)\left(\frac{a_R}{a_0}\right)^2\right\}+\tfrac{1}{2}\gamma(\gamma+1)\{1-\tfrac{1}{2}(\gamma-1)\,V\}^{2\gamma/(\gamma-1)} \times\left\{\frac{u'}{1-\tfrac{1}{2}(\gamma-1)\,V}\right\}^2+\dots,$$

and V is given by

$$a_R\frac{dV}{dt} = \frac{P_R}{\sigma}\{1-\tfrac{1}{2}(\gamma-1)\,V\}^{2\gamma/(\gamma-1)},$$

which has the solution

$$x = \frac{2}{\gamma+1}\frac{\sigma a_R^2}{P_R}[1-\{1-\tfrac{1}{2}(\gamma+1)\,V\}\{1-\tfrac{1}{2}(\gamma-1)\,V\}^{-(\gamma+1)/(\gamma-1)}]. \tag{5}$$

It may be expected that a gun tunnel will be operated to achieve piston velocities which are at least supersonic with respect to the test gas, and at least nearly sonic with respect to ambient conditions in the driver gas, whence $P_0/P_R \ll 1$. Again, provided V does not substantially exceed U, the assumption that u' is small may be sustained, whence also $u'/\{1-\tfrac{1}{2}(\gamma-1)\,V\} \ll 1$. Then ϵ may be neglected and, on putting

$$z = 1-\tfrac{1}{2}(\gamma-1)\,V, \tag{6}$$

equation (4) becomes

$$\frac{du'}{dz}-\frac{2\gamma}{\gamma-1}\frac{u'}{z} = \frac{2}{\gamma-1}\frac{P_0}{P_R}\left\{6{\cdot}72\left(\frac{1-z}{\gamma-1}\right)^2\left(\frac{a_R}{a_0}\right)^2+1\right\}z^{-2\gamma/(\gamma-1)},$$

which has the solution

$$u' = \frac{2}{3\gamma+1}\frac{P_0}{P_R}\left[\frac{6{\cdot}72}{(\gamma-1)^2}\left(\frac{a_R}{a_0}\right)^2\left\{-z^{-(\gamma+1)/(\gamma-1)}+\frac{3\gamma+1}{\gamma+1}z^{-1/(\gamma-1)} -\frac{3\gamma+1}{\gamma+3}z^{(\gamma-3)/(\gamma-1)}+\frac{(\gamma-1)^2}{(\gamma+1)(\gamma+3)}z^{2\gamma/(\gamma-1)}\right\}-z^{-(\gamma+1)/(\gamma-1)}+z^{2\gamma/(\gamma-1)}\right]. \tag{7}$$

In this case P_0/P_R has its exact value, calculated using the exact shock relations. Equations (7), (6) and (5) yield u' and V in terms of x, and therefore can be used to specify the variation of piston velocity with distance along the tube during the first stage of the motion, that is, until this velocity begins to approach the limiting value U. However, interest here is centred on determination of values of x_0 and u_0 for insertion into equation (3), and these may be obtained by putting the appropriate value of U for V in equation (5) and, through equation (6), in equation (7). The variation with U of these values of u_0 is shown in figure 2, where they are seen to be considerably more sensitive to changes in γ than in the speed of sound in the driver gas.

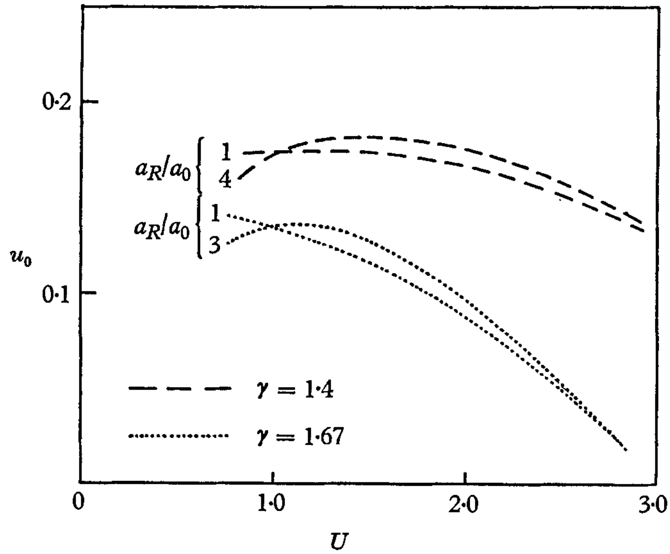

FIGURE 2. Accelerating piston—perturbation velocity at asymptotic velocity.

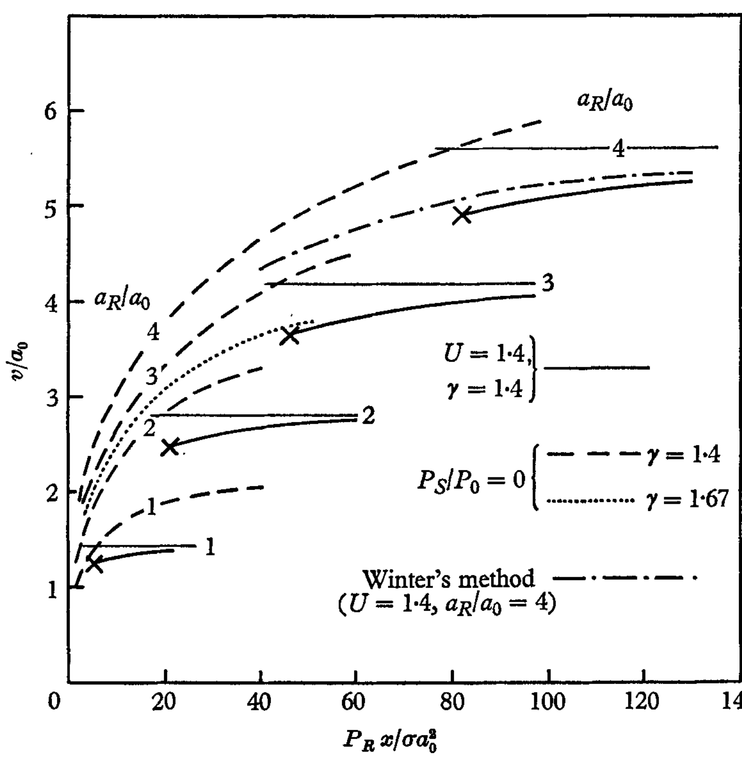

FIGURE 3. Accelerating piston—approach to asymptotic velocity U.

These values are used in figure 3 to show examples of the manner in which the piston velocity approaches U in the second, asymptotic stage of the accelerating motion. For $a_R/a_0 = 4$, the piston velocities obtained are compared with results of a numerical calculation by Winter's method, and are seen to agree satisfactorily. A similar check for $a_R/a_0 = 1$ showed slightly better agreement. It is generally not difficult to obtain values near 100 for $P_R x/\sigma a_0^2$ in gun tunnel operation, and the curves therefore indicate that, except at very high velocities, the piston velocity may be expected to come within a few percent of its asymptotic value within the length of the tube.

4. The peak pressure

As outlined by Winter, the compression of the test gas to the first, and highest, peak of pressure is accomplished essentially by a series of reflexions between the piston face and the end of the tube, of the shock wave which initially precedes the piston. These shock reflexions are evident in the photograph in figure 4, which is a time-resolved schlieren record of the motion of a nylon piston, of mass 30 mg, as it rebounds from the closed end of a 0·25 in. diameter glass tube. The background of black and white lines is caused by inhomogeneities in the glass of the tube, against which the piston trajectory appears as a dark, curved line, with the shock waves as inclined light lines. The maximum piston velocity was 1200 ft./sec. The process is represented alongside on an x–t diagram, where both the shock and the expansion waves emanating from the piston as it slows down are drawn.

At first sight, the analysis is complicated by entropy changes across the shock wave, the presence of expansion waves, and the changing pressure behind the piston as it slows down, so these factors will be considered in turn. It has already been pointed out by Evans & Evans (1956), and by Winter that the entropy rise across the shock wave becomes negligible after the third or fourth reflexion, and in fact if the dimensionless entropy change across the shock, $\Delta S/\mathscr{R}$ (where $\mathscr{R}$ is the gas constant), is plotted against initial-shock Mach number, as in figure 5 (a), it is apparent that a small error only is made in neglecting entropy changes after the first shock reflexion from the piston. An estimate of the importance of the expansion waves is obtained from figure 5 (b), in which the maximum piston velocity is compared with the minimum possible speed of sound in the compressed gas—that is, with the speed of sound after the shock wave has reflected once from the end and once from the piston. This ratio is seen to be consistently less than unity and, taking into account the further increase in the speed of sound as the peak pressure is approached, it is reasonable to assume that the expansion waves are fairly weak, and may be neglected for a first approximation. Finally, the pressure at the rear face of the piston rises to high values only as the piston velocity becomes low, i.e. towards the very end of the compression stroke. It is therefore taken to be constant at its value before shock reflexion from the piston.

Using these assumptions, an energy balance may be formulated, beginning when the piston is at 1, in figure 4, and equating the energy lost by the piston, plus the work done on the piston by the driver gas, with the energy gained by the compressed gas. Defining a cycle as being completed by the passage of the shock from the piston to the closed end and back again, then, at the end of any cycle

$$P_B(x_1 - x_{n+1}) + \tfrac{1}{2}\sigma(v_1^2 - v_{n+1}^2) = \frac{P_1 x_1}{\gamma_0 - 1}\left\{\left(\frac{P_{mn}}{P_1}\right)^{(\gamma_0-1)/\gamma_0} - 1\right\},$$

where the subscript 1 refers to conditions when the piston is at 1, the subscript $n+1$ refers to conditions at the end of the nth cycle, P_{mn} is the peak pressure developed at the closed end during this cycle, x is now the distance of the piston

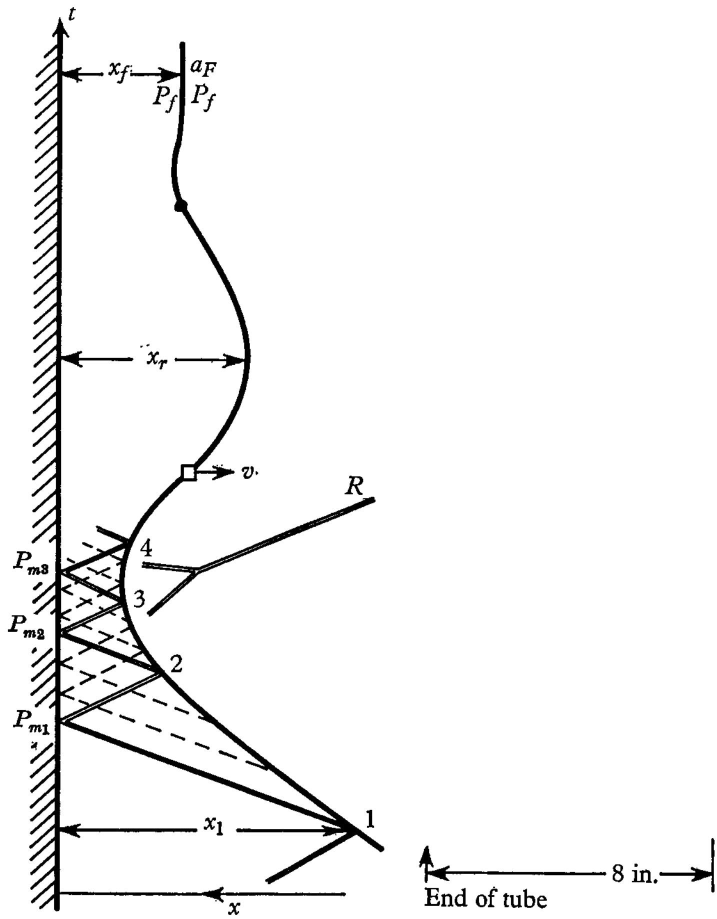

FIGURE 4. Piston motion and shock waves.

from the closed end, and γ_0 the ratio of specific heats in the test gas. It has also been assumed that P_1 is constant between the piston at 1 and the end of the tube, implying that the piston has accelerated to close to asymptotic velocity in the early stages of its motion down the tube. It is clear that maximum pressure is achieved for the smallest value of v_{n+1}^2, and, if this and x_{n+1} are regarded as being sufficiently small to be neglected, the energy balance becomes

$$P_B x_1 + \tfrac{1}{2}\sigma v_1^2 = \frac{P_1 x_1}{\gamma_0 - 1}\left\{\left(\frac{P_m}{P_1}\right)^{(\gamma_0-1)/\gamma_0} - 1\right\}, \quad (8)$$

where P_m is the peak pressure, and P_B the pressure behind the piston at 1. Utilizing the equation of state, equation (8) may be recast as

$$\frac{P_m}{P_R} = \frac{P_1}{P_0}\frac{P_0}{P_R}\left\{1+(\gamma_0-1)\frac{P_R}{P_0}\left(\frac{P_B}{P_R}\frac{P_0}{P_1}+\frac{1}{2\Lambda}\left(\frac{v_1}{a_R}\right)^2\frac{T_0}{T_1}\right)\right\}^{\gamma_0/(\gamma_0-1)}, \tag{9}$$

where $\Lambda = P_R l/\sigma a_0^2$, and l is the length of the compression tube. Following the assumption that piston velocity is close to asymptotic over most of the tube length, P_1/P_0 and T_1/T_0 may be calculated from the shock relations, and P_R/P_0 and P_B/P_R by using shock tube theory with contact surface velocity v_1. Thus specification of driver and test gases, along with the gun tunnel parameter Λ, allows determination of P_m/P_R. This is illustrated in figure 6, which shows the variation of piston velocity with Λ for fixed ratios of peak pressure to driver pressure and a range of values of the driver-gas sound speed.

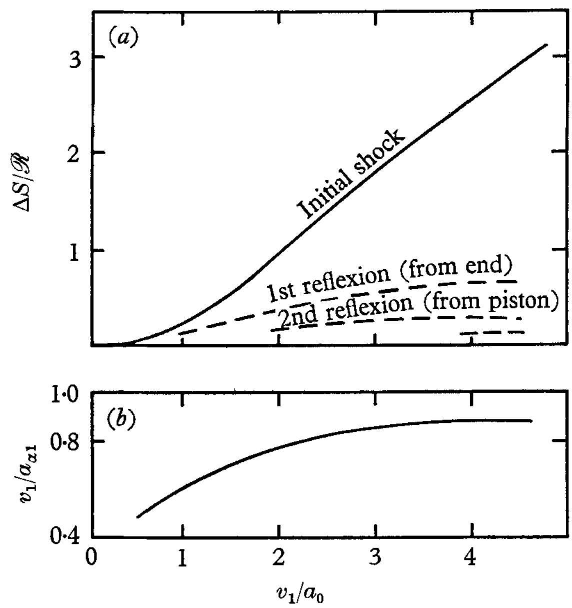

FIGURE 5. Conditions in test gas after shock reflexion; (*a*) entropy rise, (*b*) speed of sound. $a_{\alpha 1}$ = speed of sound after 1st shock reflexion from piston face; $\gamma_0 = 1{\cdot}4$.

It is apparent that high performance is obtained for high values of Λ coupled with a high speed of sound in the driver gas. To the author's knowledge, efforts made to date to improve the performance of gun tunnels have been directed at increase in either one of these two quantities with only cursory attention to the other—in fact, it is clear that for best results both must be taken into consideration. Limits of piston velocity in present practice are a little over twice the speed of sound in air and, noting that values of Λ in excess of 100 are currently being achieved, it is apparent that use of a light gas in the driver should allow almost a doubling of this velocity limit, to values approaching four times the speed of sound in air.

Of the assumptions made in obtaining equation (8), and hence figure 6, the validity of the last, concerning approach of the piston to asymptotic velocity, can be checked using the results of §2. From the remainder, the neglect of the effect of expansion waves and of the variation of pressure at the rear face of the piston are the most tenuous, and have been analysed in another paper (Stalker 1961). For the conditions of figure 6, the analysis indicates that the latter of these two effects has little influence on the peak pressure, because when the piston velocity substantially exceeds the ambient speed of sound in the driver gas, which is when the approximation of constant pressure becomes most inaccurate, the kinetic energy of the piston at 1 far exceeds the work done subsequently on the piston by the driver gas. For the former, it is found that, as the Mach number of the piston with respect to the ambient test gas increases, the error increases to

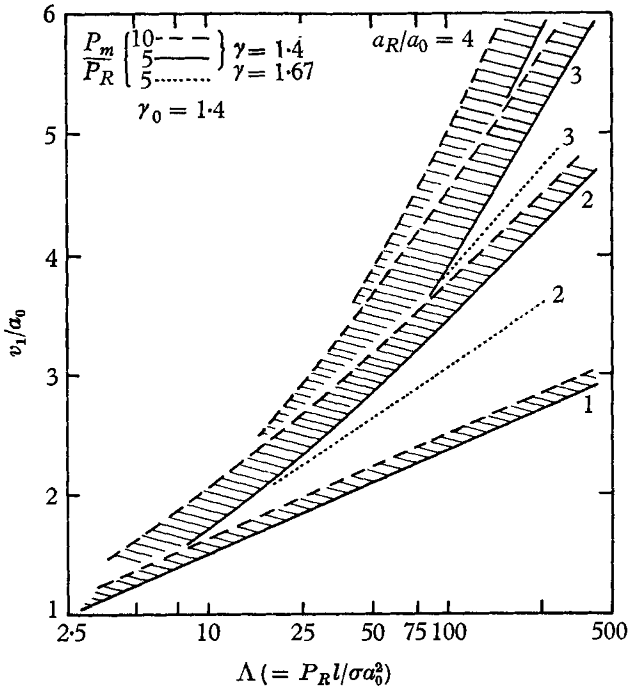

Figure 6. Performance of gun tunnel.

a limit which is reduced as the ratio of piston speed to the ambient speed of sound in the driver gas increases, and that, when these two are equal, taking account of the error approximately doubles the peak pressure. Equations (8) and (9) are therefore unsuitable for accurate calculation of peak pressure, but, since it is apparent from figure 6 that a change by a factor of two in peak pressure does not greatly affect the associated piston velocity, they are sufficient for the present study.

The high piston velocities evident in figure 6, and the associated high temperatures, bring into question the validity of the assumption of perfect gases which was made at the outset. The energy balance implicit in equation (8) can obviously be interpreted for a real gas, and, as an example, calculations have been carried out for nitrogen and oxygen, using Bernstein's tables for shock reflexion (Bernstein 1961) with a Mach number for the shock preceding the piston of 4·6, an initial pressure in the tube of 4 p.s.i. and a peak pressure of about 15,000 p.s.i. An increase of approximately 50 % over the peak pressure for perfect gas was noted, but this disappeared in accounting for the increased entropy change across the shock wave when it first reflects from the piston. These conditions represent modest pressure levels, and real-gas effects may be expected to reduce as pressures rise, leading to the conclusion that errors induced in maintaining the assumption of perfect gases are small enough to be ignored here.

5. Piston oscillations

It can be seen from figure 4 that, after achieving peak pressure in the first violent compression, the piston rebounds from the closed end and executes one or more oscillations before coming to rest. Though not a primary limitation on performance, these oscillations do imply a delay in establishing steady conditions in the test gas, and are therefore worthy of a brief, not necessarily very accurate, treatment. Such a treatment has been reported (Stalker 1961), and satisfactorily compared with experimental results. A résumé of the analysis only will be presented here.

The model used for the piston motion is shown in figure 4. The shock R, generated as the piston first reverses, is assumed to propagate at constant velocity into a region of uniform flow, conditions behind the shock are calculated by assuming a stationary piston, the velocity of the piston is assumed to be considerably lower than the speed of sound behind the shock, and the pressure is taken as constant between the piston and the end at any instant. The equation of motion for the piston is thus

$$\sigma \frac{d^2x}{dt^2} = P_f\left\{\left(\frac{x_f}{x}\right)^{\gamma_0} - \left(1+\frac{\gamma-1}{2a_F}\frac{dx}{dt}\right)^{2\gamma/(\gamma-1)}\right\},$$

where subscript f refers to conditions in the test gas when the piston is stationary, and a_F is then the speed of sound in the driver gas behind the piston. Neglecting powers of $a_F^{-1}dx/dt$ higher than the first, we have

$$\frac{\sigma}{P_f}\frac{d^2x}{dt^2} + \frac{\gamma}{a_F}\frac{dx}{dt} = \left(\frac{x_f}{x}\right)^{\gamma_0} - 1. \tag{10}$$

The maximum speed v' of the piston as it first rebounds from the end to begin its series of oscillations, and the amplitude x_r of this first half-cycle, may be obtained to reasonable accuracy by neglecting the damping term in equation (10). Then, noting that damping is most effective when piston velocities are highest, and that this occurs for values of x near x_f, the right-hand side of equation (10) is linearized to yield

$$\frac{\sigma}{P_f}\frac{d^2x}{dt^2} + \frac{\gamma}{a_F}\frac{dx}{dt} = \gamma_0\left(\frac{x_f}{x} - 1\right), \tag{11}$$

from which is obtained a damping ratio, comparing successive maxima of the piston velocity as it passes through $x = x_f$, i.e.

$$\frac{v'}{v''} = \frac{v''}{v'''} = \frac{v'''}{v^{\mathrm{iv}}} = \ldots = \exp\left[\pi\left\{4\frac{\sigma\gamma_0}{P_f x_f}\left(\frac{a_F}{\gamma}\right)^2 - 1\right\}^{-\frac{1}{2}}\right]. \tag{12}$$

This enables a calculation of the number of oscillations required to reduce the piston velocity to a value (say 100 ft./sec) at which friction may be expected to render the piston stationary. Using figure 6 to obtain conditions corresponding to $P_m/P_R = 5$, the damping ratio for piston velocities of interest is found to be fairly insensitive to a_R, changing from about 3·4 to 4·1 as v_1/a_R increases from 1·0 to 2·0 with $\gamma = 1{\cdot}4$, and varying by less than 15 % at a fixed value of v_1/a_R within this range. Values with $\gamma = 1{\cdot}67$ were about the same at $v_1/a_R = 1$, but remained roughly constant within the range. Accordingly, the piston velocity may be expected to be reduced to one-tenth of its initial maximum rebound value in one cycle. Bearing in mind that values of v' are unlikely to exceed 4000 ft./sec, it is clear that the first cycle of the oscillations is the most important, since, after this, friction effects may be expected to become significant, leading to enhanced damping of the motion.

An estimate of the period of the first half-cycle is obtained by noting that, from this point of view, the most important part of the motion occurs when velocities are lowest, that is, for values of x near x_r. The right-hand side of equation (11) is therefore replaced by a linear term yielding correct values at $x = x_r$ and $x = x_f$ leading to

$$\frac{\sigma}{P_f}\frac{d^2x}{dt^2}+\frac{\gamma}{a_F}\frac{dx}{dt}+\alpha\left(\frac{x}{x_f}-1\right) = 0, \tag{13}$$

where $\alpha = \{1-(x_f/x_r)^{\gamma_0}\}/\{(x_r/x_f)-1\}$.

The system described by this equation has a half-cycle period given by

$$\tau = \frac{2\pi\sigma a_F}{\gamma P_f}\left\{4\alpha\frac{\sigma}{P_f x_f}\left(\frac{a_F}{\gamma}\right)^2-1\right\}^{-\frac{1}{2}}. \tag{14}$$

Once again, figure 6 is used to obtain conditions corresponding to $P_m/P_R = 5$, and, using l/a_R as a reference time interval, it is found that, when the ratio $\tau a_R/l$ is plotted against $\Lambda(a_0/a_R)^2$ for values of a_R/a_0 from 1 to 4 with $\gamma = 1{\cdot}4$, the variation falls within the cross-hatched zone shown on figure 7. It will also be seen that this variation is not very different when $\gamma = 1{\cdot}67$, whence, for the purposes of present discussion, $\tau a_R/l$ may be taken to depend only on the parameter $\Lambda(a_0/a_R)^2$.

It has already been established that high values of Λ are essential for high performance, and it becomes apparent from figure 7 that they may also be required to minimize the delay in arriving at steady conditions at the end of the tube. From the figure, only insubstantial gains in this direction are realized in increasing values of $\Lambda\ (a_0/a_R)^2$ above 15 but, although such values are easy to achieve when $a_R = a_0$, difficulty may be expected when light gas drivers are used. This is of particular importance when the 'pressure plateau' mode of gun tunnel operation is preferred, in which testing is conducted during the period taken for the shock R formed at the first piston reversal to propagate to the diaphragm station and reflect a disturbance back to the end of the tube. The time available in this method is roughly twice l/a_R, wherefore, were hydrogen being used as driver and air as test gas, figure 7 indicates that, with $\Lambda = 100$, $2\tau a_R/l = 1{\cdot}6$, and almost all the available test time is taken up by piston oscillations. However, it will be observed that if a somewhat reduced performance is acceptable, and nitrogen is mixed with the hydrogen to halve the value of a_R, the relative period of the oscillations may be halved. Another remedy lies in the 'tailored piston' technique (East & Pennelegion 1961) wherein piston oscillations are eliminated by arranging that the peak pressure is, at least approximately, equal to the pressure P_f behind the piston when it becomes stationary. As an example, it can be calculated from equation (9) that, with $a_R/a_0 = 2$, a piston velocity of $3{\cdot}2a_0$ may be achieved with $\Lambda = 500$. Whilst this at first sight may seem to be a large value for the gun tunnel parameter, it should be remembered that the technique permits very high driver pressures, making such values possible.

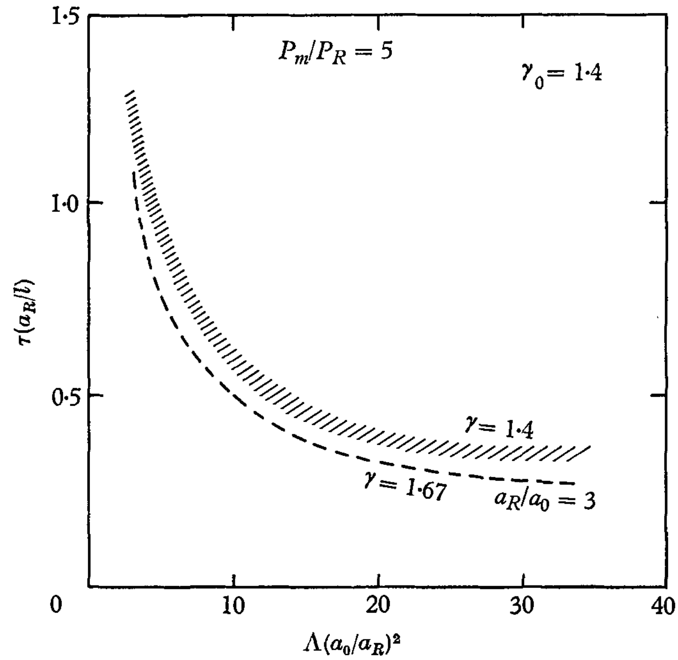

FIGURE 7. Half-cycle period.

6. Conclusion

The performance of a gun tunnel is primarily limited by its capacity to withstand the peak pressure produced in first reversing the piston motion. Taking the view that it will in general be possible to design the compression tube at the closed end for a transient peak pressure a few times that of the driver, a value of $P_m/P_R = 5$ has been used as required as a basis for calculation in this study. Of course, any other reasonable value could be used, in which case qualitatively similar conclusions would be expected. Results have emphasized the fundamental importance of the gun tunnel parameter, $\Lambda = P_R l/\sigma a_0^2$, not only in governing the accelerating phase of piston motion (in agreement with previous authors, e.g. Winter 1960) but also, in conjunction with the driver-test gas ratio of sound speeds, in determining the maximum allowable piston velocity, and the delay occasioned by waiting for subsidence of the following piston oscillations.

Study of the accelerating phase has indicated that, with values of Λ achieved in current practice, it is reasonable to use the asymptotic value of piston velocity in calculating peak pressure, at least until this velocity exceeds about four times the speed of sound in air. Then, developing a theory for peak pressure, it is found that, by combining high values of Λ with an increased speed of sound in the driver gas, piston velocities of roughly four times the speed of sound in air might be achieved. This theory may, in certain circumstances, underestimate the peak pressure by as much as a factor of two, but, because the peak pressure at high values is very sensitive to piston velocity, it has been possible to define allowable velocities reasonably closely. Finally, a rudimentary consideration of the oscillating phase of the piston motion suggests that, although the oscillations are damped fairly rapidly, values of Λ are such that the period of oscillation may perhaps preclude the 'pressure plateau' mode of operation with high driver sound speeds.

Concluding, mention should be made of the relative dimensions to be desired in the compression tube. To achieve large values of Λ with given pressures, it is clearly necessary to maximize l and minimize σ, the piston mass per unit area. In practice, it has generally been found necessary to use a skirt length roughly $\frac{2}{5}$ of the piston diameter, to ensure that the piston remains upright in the tube during operation. For a given material and piston configuration, this determines the thickness of the piston, and thus its mass per unit area, wherefore Λ reduces with diameter, and high piston velocities are favoured by a large length/diameter ratio of the compression tube. Inspection of figure 6 will indicate that with present values of Λ (i.e. approximately 100) the potentialities of light gas drivers are not fully exploited, suggesting that the use of compression tubes with larger length/diameter ratios might well be explored experimentally.

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